Rational choice, neuroeconomy and mixed emotions

Pierre Livet*

CEPERC, University of Provence, 29 Avenue Robert Schuman, 13621 Aix en Provence Cedex 1, France

Experimental psychology has shown differences between predictions of theory of decision and human choices. Emotions like regret can partly explain these differences. Neuroimagery used in combination with behavioural economics (neuroeconomics) has been used in order to try to disentangle the different emotional and rational factors (regret, rejoicing, reward, costs, uncertainty, trade-off between positive and negative aspects of different options). Emotions then appear as much more complex and mixed affective states than usually assumed. Not only might we feel a positive affect in punishing unfair partners, but mixed emotions can, for example, combine transmutation of previous anxiety into relief and elation by comparison with another less exciting option (elating relief). At the level of complexity of these mixed emotions—which we formally represent by comparisons between ‘unexpected utilities’ and expected ones—the main biases that Kahnemann and Tversky have shown can be explained. In spite of the complexity of these mixed emotions, some of these hypotheses might be partially tested by brain imagery.

Keywords: emotion; neuroeconomics; brain imagery; regret; unexpected utilities; elating relief

1. INTRODUCTION

Although research on emotions and rationality is relatively recent, it has evolved and this evolution is informative. The impetus was provided by two sets of results. On the one hand, Kahneman & Tversky (1979, 2000) and their colleagues showed that our choices under experimental conditions differ strongly in some circumstances from what rational choice theory suggests: for example, we prefer a smaller certain gain to an uncertain one but with higher expected utility. On the other hand, Bechara et al. (1994) argued that patients impaired in regions of the prefrontal cortex related to emotional sensitivity are not able to take into account, in their choices between two stacks of cards, long-term anticipations about the fact that choosing the cards from the stack with more frequent but moderate gains is better than choosing from the stack offering a combination of less frequent higher gains and losses (Iowa Gambling Test). This is not to say that being emotionally impaired is always bad for choices in uncertainty. Recently, Shiv et al. (2005a,b) showed that in some circumstances, emotionally abnormal patients make more rational choices, being insensitive to emotional bias. Nevertheless, these new perspectives combine to suggest that the lack of realism in the rational choice theory may be the result of having neglected the role of emotions. Elster (1996, 1999) drew, from this relationship between psychology and neurosciences, an explanation of our decisions: they can be motivated either by selfish interest, by public reason or by emotions.

In what follows, we will first give a brief overview of the developments in the research on emotions and decision-making, in psychology, economics and neurosciences, concluding that emotions are not as simple as first supposed. Second, we will show that the biases of our decisions can be explained by two mixed emotions, elating relief and prudential pride (in two dual situations: elation when we get a bigger but riskier gain, decrease in anxiety when we prefer a less risky but smaller gain), a hypothesis that is a challenge for neuroimagery.

2. A SHORT HISTORY OF THE RESEARCH ON EMOTIONS AND DECISION-MAKING

At first, economists reacted to the first set of experimental results by conceiving and finding variations in the formalism of expected utility that could deal with the conflict between the previous models and these experiments. We are less sensitive to medium-sized probabilities than to small ones and to certainty. Non-additive probabilities and Choquet integral—a cumulative function between 0 and 1 that allows our sensitivity to grow with probabilities in a nonlinear way—can solve this problem (Gilboa & Schmeidler 1989; Schmeidler 1989). Meanwhile, other economists tried to give an emotional interpretation to these biases. In particular, Loones & Sugden (1982) put forward a theory of regret. According to them, our choices are mainly framed by the emotions raised by our comparison between one possible outcome of the supposedly chosen lottery and another possible outcome of the other and compared lottery: regret (or rejoicing), when the difference is due to our choice, or disappointment, if the difference is due to chance. We weight each ‘emotional’ difference by its probability and calculate the weighted differences in order to make our choice. The

* pierre.livet@univ-provence.fr

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predictions of this theory were tested by experiments and, unfortunately, have failed in a few of them. Another problem was that emotions were only attributed to the difference between outcomes (gains or losses). But differences between probabilities can also reasonably be assumed to raise emotions: if one branch of a tree representing a lottery gets the desired outcome with a smaller probability than the branch of the compared lottery, we feel more anxious when choosing the first one. Sugden may not have taken into account how pervasive emotions are in our appraisal of the features of situations. Nevertheless, we will see that his distinction between regret (owing to our own actions and choices) and disappointment (owing to the results of chance) has been proved to be anchored in neurological differences (Camille et al. 2004; Coricelli et al. 2005).²

Next an explanation of the differences between real (or experimental) choices and the theory of rational choice was sought in purely psychological terms. For example, experimental researchers found that people who are sad try to compensate for the loss related to their sadness, while anxious or fearful people prefer to reduce uncertainty. ‘Incidental sadness increased tendencies to favour high-risk–high-reward options. Incidental anxiety, in contrast, increased tendencies to favour low-risk–low-reward options’ (Loewenstein & Lerner 2003, quoting results of Smith & Ellsworth (1985) and Ragunathan & Pham (1999)). Here, ‘incidental’ means experimentally induced, mainly by sad pictures or alarming stories. Lerner & Keltnner (2000, 2001) showed that a similarly manipulated emotional state of anger increases tendencies to be less sensitive to risk, in a way similar to the effect of being in an optimistic mood. By contrast, people in a state of fear are more sensitive to risk, as are pessimistic people—and these pessimistic people are more accurate in their estimation of risk, if we take the rational choice theory as the ideal reference for choice. Other experiments tend to show that we are not very reliable in intertemporal evaluations of our reactions and choices in emotional states that differ from our present state. People in a ‘hot’ state (e.g. angry) are not good predictors of their choice in a cold state (not angry) (Loewenstein et al. 1998). In general, people feel that they will be unable to deal with possible problems in the future even though these are similar in difficulty to problems that they have overcome in the past. All this research, in relation to Loomes and Sugden’s theory, focuses on emotions that are the results of comparison between a real situation and a counterfactual one, or between two currently counterfactual situations, only one of which is realizable in the future. These emotions imply that a comparison between real present state and possible but counterfactual future state raises present emotions and that a comparison between two counterfactual states has similar present effects. In what follows, we will consider anticipatory and comparative emotions for these two categories of affects. As an example, the 1992 Olympic games bronze medallists displayed on average a more positive affect when they received their medals (they were asked to rank their satisfaction in a qualitative order) than did silver medallists (Medvec et al. 1995). The explanation could be that the two kinds of medallists were comparing their situations with their two nearest situations: the first, comparing his situation with the counterfactual situations of having coming ‘fourth’ and not getting medals, felt an elating relief, and comparing it with the counterfactual situation of getting silver, rightly saw missing the gold medal as disappointing, while the second compared it mainly to the situations of winning a gold medal (or a bronze medal). In addition, silver medallists underestimated the satisfaction of bronze medallists because in their eyes, bronze now represented a down-grading.

These studies have paved the way for the field of economics called ‘behavioural economics’ (Glimcher et al. 2009). As Rubinstein (2006) mentions, the motivation of agents in behavioural economics need not be rational. Economists still use assumptions from diverse sources in order to model and explain these behaviours, some of them inspired by theories of economics (mainly the game theory) and some of them based on a psychological—and frequently an emotional—interpretation. One of Rubinstein’s (2008) criticisms is that a psychological interpretation confuses the utility function—depending on the order of preferences and the probabilities—with the evaluation of welfare; we can act against our own welfare and be perfectly rational if rationality simply consists of maximizing our expected utility consistent with our preferences and the probabilities, as assumed by the rational choice theory.

The question arises then whether this confusion between preferences or utility and welfare (or desires for well-being, which are related to emotions, because emotions are reactions to changes of situation that are relevant to the satisfaction of our desires) constitutes an error, or whether it stems from a real relationship between the reasons for our choices and our desires and emotions. If the second hypothesis were true, then the abstract notion of utility—related to the order of revealed preferences—of the rational choice theory would have to be fleshed out by desires and emotions. If there are discrepancies between the formal theory and the formal models that describe the processes of choice for an agent endowed with desires and emotions (Loewenstein et al. 2008), then the second kind of formalism should be used in preference to the classical theory, and economists should have to integrate these models into a new coherent axiomatic framework.

3. EXPERIMENTAL EVIDENCE ON EMOTIONS
In order to show that choices imply desires and emotions, experimental psychology is not sufficient, for its categorizations of states of mind into emotions rely mainly on the introspective examination of subjects, and such categorizations could be suspected of being culturally dependent. Another way to justify the assumption that rationality of choice implies emotions—the dynamics of which cannot be supposed to be reducible to the axioms of the rational choice theory—is to correlate activities of choice with cerebral activities, and to show that these cerebral activities involve activating regions of the brain whose activation has been shown to be correlated with emotions in studies not dependent on situations of choice. This kind of research is the basis of neuroeconomics and
implies a branching-out of neuroeconomics from behavioural economics.

Damasio (1994), Le Doux (1996) and other researchers have already studied cerebral activities related to emotions, either by describing abnormal emotional behaviour in patients damaged in some region of their brain, or by studying the hormones secreted and diffused in the brain and the organism when people and animals are supposed to be experiencing emotions. Their experimental protocols rely mainly on distinguishing between simple or ‘basic’ emotions like fear, anger, happiness, sadness, disgust and surprise. Ekman (1984) has shown that people from very different cultures put together, in the same five classes, faces that are supposed to express each one of these five distinct emotions. Damasio et al. (2000) have shown that reactions to pictures and stories assumed to induce each of these emotions could each be related to a different specific network of activated brain locations (scattered throughout the brain, and partly shared), giving neuronal correlation to the psychological distinctions made by Ekman.

This new neuroeconomics used functional magnetic resonance imagery or positron emission tomography to try to find correlations between affects in choice and neuronal activity locations identified by brain imagery research as related to different emotions. The methodological framework of such research was not very well defined, but it was roughly the following: if we have already observed a correlation between both emotional activity and neuronal activity in some region of the brain, and find this region activated during an experiment involving choice activity, we can argue that this situation of choice, and the activity of the mind during the choice, imply an activation of this location of the brain, and thus the correlated emotional activity, which can be confirmed by measuring skin resistance. As situations of choice imply reactions to outcomes and to uncertainty and no ‘basic’ emotion is strictly related to uncertainty or outcomes, we have to relate uncertainty to anxiety and fear, and outcomes to satisfaction or happiness if they are gains and to sadness if they are losses.

In addition, neuroeconomics has sought the location in the brain of more socially oriented interaction, mainly fairness or sensitivity to unfair behaviour in an interaction. A combination of behavioural economics and neuroeconomics has been proposed, centring on the experimental paradigms using variations in games in which reciprocation, implying either sensitivity to equity or trust, is possible but not mandatory, like the ultimatum game and the trust or investor game. In the first, one agent is given a sum of money and can choose to give another person a part of this money. If the other accepts, the money is divided as proposed. If he refuses, no money is given. Most agents refuse a share smaller than 25 per cent, despite the fact it would be ‘rational’ for them to accept any offered ratio. In the trust game (Fehr et al. 2005), money is given to an investor, who chooses how much of this money he gives to another person (say, a producer), who has to choose in turn how much of the common gain (the investment is assumed to treble the sum) to give back to the investor. This game is played successively with different players.

In some versions, the investor or even a third party, an observer, is offered the opportunity to punish the producer when his returns are judged to be too small, but at a cost to the punishers (Fehr & Gächter 2002).

In another approach, attention is focused on hormonal factors, mainly oxytocin and serotonin. Zak et al. (2005) have shown that in the trust game, the blood of investors with the intention to trust (the intention of transferring money to the producer) presents a higher level of oxytocin than the blood of people with the opposite intention. The producer receiving a transfer sufficient to signal trust presents also a higher level of oxytocin. But a higher level of oxytocin in the investor does not predict a higher amount of transferred money. Zak et al. suggest that oxytocin is related to the presence of trust, to signals of trust, but is not the cause that induces the level of transfer: a higher level of oxytocin when transfers have been made does not imply a higher level of trust. Baumgartner et al. (2008) contrast injections of placebo or oxytocin in the investor, and two phases, before and after receiving feedback about how much the producer has paid him back. The game was played either with a human agent or with a computer. Surprisingly, oxytocin had no effect (in comparison with placebo) on the investor during the pre-feedback period. Subjects receiving a placebo decreased their trusting behaviour after negative feedback (the partner did not pay back) in 50 per cent of cases, whereas subjects receiving oxytocin did not change their behaviour in this post-feedback phase. In the game with the computer, agents showed no change in response to the feedback information: they interpreted the behaviour of the computer as a random signal conveying no information that could be used to predict future behaviour. The response time of subjects with oxytocin was considerably shorter than that of the patients receiving a placebo. These results could be compatible with different interpretations. Baumgartner suggests that placebo subjects in the pre-feedback phase were still interested in testing the reciprocal trustworthiness of their partners and could only test it by transferring the money, which implies, in this first phase, trusting their partners, so that they did not differ from oxytocin subjects. Oxytocin is correlated to a reduction in fear of social betrayal. We could add that if oxytocin is a response to trust and not a predictor of the intensity of trust behaviour, the oxytocin effect must not be comparatively significant in the pre-feedback period, when no signal of trust has been given by the partner. In contrast, in the post-feedback period, subjects have to respond to the given signals. The conclusion could be that oxytocin has effects on the reaction to signals from social partners, and not on your own state of trust. Of course in interactions, a private state of trust and received signals of trust usually do not need to be disambiguated. Notice that in this experiment, the placebo subjects exhibit higher activation in the dorsal anterior cingulate cortex, a brain region frequently implicated in conflict monitoring: here, oxytocin reduces the need to use more cognitively sophisticated evaluation of conflicting situations.

In these studies, the relationship between decision and emotions is not straightforward, apart from the
fact that oxytocin is supposed to improve reactions in situations associated with social anxiety and stress. Oxytocin subjects in Baumgartner's experiment presented a reduction in activation in the amygdala, so that Baumgartner relates oxytocin to a reduction in fear. But, while feeling that someone is trustworthy must reduce anxiety, social trustworthiness cannot simply be equated with decrease in fear—we can fear that the reciprocal action of our partners will fail and still regard them as trustworthy.

Other researchers work, not on hormonal diffusion, but rather on neuronal activation. Sanfey et al. (2003) have shown that the reaction to very unequal offers in the ultimatum game is correlated to brain activation in the bilateral anterior insula, usually related to negative emotional states, and in the dorsolateral prefrontal cortex (DLPFC), related to goal maintenance and executive control. The interpretation could be that the agent's affective evaluation is negative, so that he has to decide between his wish to reject this bad offer and his wish to get some money. In support of this scenario, the activation in the dorsolateral prefrontal cortex is greater for people accepting the unfair proposal, while activation in the insula is greater for people rejecting it.

De Quervain et al. (2004) introduced into the trust game the opportunity for the investor to punish the producer. They observed that punishment activates the caudate nucleus, associated with the processing of rewards. Baumgartner also observes activation in the caudate during interaction with the morally neutral partner, and deactivation with good and trustworthy partners. These activations seem to imply not only reward, but the need to decide in an uncertain situation between actions with conflicting affective impacts. When punishing is costly, other regions related to the processing of the integration of criteria with conflicting implications on choice, mainly the ventromedial prefrontal cortex, are activated, whereas if punishing is free, they are not activated. Knoch et al. (2006) disturbed the dorsolateral prefrontal cortex by low-frequency repetitive transcranial magnetic stimulation during the ultimatum game. This region is supposed to be associated with executive and top-down control and inhibition or overriding of different motives. Surprisingly, disruption of the left prefrontal cortex did not reduce rejections of partners' unfair offers, whereas disruption of the right dorsolateral prefrontal cortex has this effect.

All these experimental studies were aimed at finding a correlation between brain activation and emotionally charged situations. The categorizations of emotions supposed to be induced by the situations were our supposedly 'basic' ones: fear, anger, anxiety and the feeling of being more secure with trustworthy people. We observe that, in fact, emotional situations also activate regions related to control, inhibition and monitoring of conflicts. We will develop more on this theme further pointing out that neuroimagery can also be used to test our emotional intuitive categorizations. For example, Coricelli et al. (2004, 2005) designed experimental situations in which people could either compare their anticipation of a gain in a lottery with the gain obtained in one draw of the other lottery, or compare the obtained gain in the chosen lottery with that obtained in the lottery not chosen. In the first case, people should feel disappointment, as they were not responsible for the random draw of the lottery, while in the second case, they felt responsible for having chosen their lottery, so that they felt regret when the gain of the other lottery was higher. Disappointing situations activate the substantia nigra, related to anticipation of rewards, while situations causing regret activate both the amygdala, related to fear but also in general to the affective evaluation of new situations, and the orbitofrontal cortex, related to absorption and integration of different information, possibly in conflict. Both disappointing situations and situations of regret activate the striatum, related to rewards. The striatum is activated by gains, and deactivated by losses.

Such a study shows both that distinctions drawn between emotions (the distinction between disappointment and regret) can be confirmed by neurological analysis and that regret, related to responsibility and choice, activates regions other than those associated with pure affective evaluation, mainly regions also associated with the control and integration of different and perhaps conflicting information. On the one hand, it shows that neuroimagery can help separate emotional states into distinct categories, consistent with our intuitive conceptual elaboration on the classification of emotions, and on the other, it alerts us to the fact that we cannot consider emotions only in affective terms, but that they are closely linked to action, the control and monitoring of execution and the processing of conflict between features that lead to contradictory evaluations.

The current research may lead to the following opinion: emotions cannot be pure and simple ('basic') states of the mind, separated from each other and from other non-affective cognitive processes. Most of our emotions are affective states that mix different evaluative aspects (gain or loss and irresponsibility, for disappointment; gain or loss and responsibility, for rejoicing or regret), and that mix affective aspects with the control and integrative aspects of action and cognition. McRae et al. (2008) show that intense emotions trigger activations in regions related to the processing of conflicts and the allocation of attention, such as the dorsal anterior cingulate cortex. They show that these activations are arousal dependent.

A meta-analysis of 162 experiments in this field reinforces this appraisal of the tendencies of the current research trends. Köber et al. (2008) conclude that the regions of the brain shown to be activated in most of the studies using strict methodology are regions correlated to physiological regulation (vagal system), generation and inhibition of action, retrieval and processing of memories and perceptual processing. The activations of the cingulate region imply both affective and cognitive processes, or mixed ones, such as allocation of spatial attention to emotional stimuli in the posterior cingulate and memory-guided representation of context (emotions are strongly linked with memory: being in a given emotional mood triggers preferentially emotions of the same mood). They also involve regions associated...
with the ‘theory of mind’ (representations of the intentions, emotions and representations of other people. Of course, emotions are related to activation in regions associated with reward, positive or negative): the striatum and the insula. But iterative cluster analysis reveals a relationship between these reward regions and the lateral frontal cortex, and a connection with cognitive and motor systems. As mentioned, the amygdala is not only associated with fear and disgust, but its activation can be related to affective significance and response to novelty: it marks stimuli as being of unknown or uncertain predictive value and increases attention. Iterative cluster analysis also reveals a connection between global networks. The medial prefrontal cortex group (related to regulation of emotions, e.g. by inhibition) is not directly related to the cognitive/motor group, but only through the mediation to the dorsal and posterior insula regions (the ‘paralimbic group’), which can be interpreted as a stronger relationship with core affective response and visceromotor reactions rather than with motor execution. This could imply that Frijda’s (1986) theory of emotion as ‘readiness potential for action’ places the focus on an aspect that is not the central one.

In this meta-analysis, Kober et al. criticize the tendency of the previous research to take for granted ‘intuitive’ categories of emotion and to underestimate how intricately emotions are linked with other functions. Trying to match neuronal activations with ‘basic’ emotions is not a reliable paradigm. However, Kober et al. themselves rely heavily on other kinds of categorizations (e.g. allocation of spatial attention, retrieval of memories) that are also based on our intuitive categorizations. Unfortunately, we cannot dispense with these ‘intuitive’ or socially shared categories. But this criticism does not alter the conclusion that neuroimagery enables us to compare these different categories with each other and become sensitive to their interdependency.

This overall trend for research on emotions and decision-making to take into account the fact that no emotion is pure and is only an affective state of mind is opening the way for mixed emotions to be studied (mixing different valences or affective aspects, like anxiety and attraction to a gain), as well as cognitively complex emotions (like anticipatory emotions, triggered by comparisons with counterfactual situations); ‘pure’ and ‘basic’ emotions do not exist except as simplifications, while real emotions are mixed with anticipation and comparisons with counterfactuals.

Psychological research on emotions has already shown (e.g. in Larsen et al. 2004) that feelings of happiness are tempered and modulated by our comparisons, as are feelings of disappointment and regret. But it has also been shown that in child development, emotions based on comparisons and counterfactuals, and in general mixed emotions (like nostalgia) are acquired later. Even in adults, emotions related to counterfactuals appear late, and negative counterfactual emotions last longer. One emotional state can mix negative and positive affects. Emotions do not show univariate bipolarity, but rather multivariate bipolarity.

In what follows, we take advantage of the increased complexity of the research on emotions to focus on mixed and multivariate emotions, attempting to show that mixed and counterfactual emotions are used in our deliberative choices, and can explain the ‘biases’ supposed to be inconsistent in Kahneman and Tversky experiments, re-establishing their coherence. Finding neuronal correlates—probably not local regions but specific networks—for these emotions is a challenge that the neurological research on emotions is now able to meet.

4. TWO DUAL MIXED EMOTIONS: TOWARDS AN EMOTIONAL RATIONALITY

We cannot expect strong or ‘hot’ occurrent emotions to be particularly in tune with deliberative rationality. At best, anger, fear, disgust and the like can be assumed to be evolutionarily best responses to extreme cases in which our survival or the chances of reproduction of our genes are at stake. They are related to brute and rapid heuristics for instant decision-making. Rationality of choice, on the other hand, is not related to these rough and brute heuristics, but rather to cool deliberation, when we have time to make our decision in advance. Are there still emotions in the cool situation of deliberation?

Some of the psychological and neuroimagery studies mentioned before assume that we feel emotions in advance, when imagining what might be the outcome of a future action. These emotions are called anticipatory emotions. What are the similarities and differences between ordinary emotions and anticipatory emotions? Ordinary emotions are raised by a present event, but both imply an occasion— in anticipatory emotions, it is the present emotional effect of our anticipatory representation. In my approach, every occurrent emotion is raised by the perception of a differential between the trend of my previous routine, in my situation before the emotional event and one feature of the new situation, appraised by reference to my desires and preferences.

Anticipatory emotions imply what could be called a longitudinal comparison (usually implicit) between our present situation and the future outcome. In deliberation, anticipatory emotions are raised not only by this longitudinal comparison, but also by a lateral one, a comparison between two outcomes: one outcome of a branch of one first lottery and another outcome of a branch of another compared lottery (lotteries are used here because most of our future possible states are uncertain). We compare the difference between the gains (or losses) from these two branches. This difference (taking the anticipation of one outcome as our reference state, and perceiving the differential between it and the other outcome) is one source of our comparative anticipatory emotion.

As the results of a future action are usually uncertain, anxiety may be another source of anticipatory emotions. We have to take into account these uncertainty or anxiety aspects in comparative emotions as well. We compare the two risks or uncertainties of the two branches (in this paper, we will simplify the problem by taking uncertainties as probabilistic risks, and probabilities as related to anxieties, a disputable but convenient simplification). Our emotion is now raised by the differential between the uncertainty of our
Let us take very simple lotteries. Let $G_1^{\text{max}}$ and $G_1^{\text{min}}$ be the two gains of lottery $L_1$, with respective probabilities $p_1^{\text{max}}$ and $p_1^{\text{min}} = 1 - p_1^{\text{max}}$, and $G_2^{\text{max}}$ and $G_2^{\text{min}}$, the gains of lottery $L_2$, with respective probabilities $p_2^{\text{max}}$ and $p_2^{\text{min}} = 1 - p_2^{\text{max}}$. Suppose $G_1^{\text{max}} > G_2^{\text{max}}$, $G_1^{\text{min}} > G_2^{\text{min}}$, $G_1^{\text{max}} > G_2^{\text{min}}$, $G_1^{\text{min}} > G_2^{\text{max}}$ and $p_1^{\text{max}} < p_2^{\text{max}}$. In several experiments by Kahnemann and Tversky, $G_1^{\text{min}}$ and $G_2^{\text{min}}$ are 0, as in one lottery of Allais’ paradox.

For gains, comparative anticipatory emotions in which uncertainty and outcomes intercede in opposite directions are dual emotions of two kinds.

The first kind of emotion is raised by the fact that a branch of lottery $L_1$ gives a higher gain (e.g. $G_1^{\text{max}}$) than that of a branch of lottery $L_2$ (e.g. $G_2^{\text{max}}$), but with a smaller probability. We imagine that we are in the situation in which we get this higher gain $G_1^{\text{max}}$, and our mixed comparative emotion is raised by the part of the gain that was not expected, as we have said. The first component is positive (elation). Since the gain was risky, when we imagine ourselves getting the gain, we feel intense relief in comparison with our previous anxiety. In a sense, our previous anxiety is transmuted into relief when we get the gain. At the same time, we feel more excited when we get a bigger gain and that was much more unlikely than the possible minor one. We feel more favoured by fortune, we feel elated because we took a risk instead of being cautious and were rewarded. Elation implies comparing the unexpected part of our gain to the expected gains—from the chosen lottery and from the other lottery. The second is a negative one: we also have to count the burden of our previous higher anxiety in comparison with the anxiety raised by the branch of the other lottery. As elation, the burden of anxiety is not raised here by the expected utility, but by the anxiety of missing the unexpected surplus of this gain; we will call the anxiety that is additional to the anxiety implied by the expected negative utility of missing it additional anxiety. We will call the emotion that combines elation and additional anxiety elating relief: elation is related to the gain not expected; we subtract additional anxiety. The remaining part is elating relief: if elation is bigger than additional anxiety, anxiety is transformed into relief.

The second kind of emotion is the dual of the first: it is raised, for example, by the anticipation of the gain $G_2^{\text{max}}$ that has a higher probability, even if it is smaller than $G_1^{\text{max}}$. We can call it prudential pride: we are proud not to have taken higher risks, even at the cost of giving up the possibility of getting a bigger gain. Notice that these two emotions have an intertemporal dimension. Elating relief implies that we feel more anxious before knowing the result of the lottery, but hope to exchange this for elation after the result—Elster might say that the elation is the transmutation of the previous anxiety. Prudential pride, in contrast, implies preferring to be less anxious before knowing the result, in exchange for being less happy when the result occurs—and when the bigger gain is drawn in the other lottery.

How can we give a formal representation of these two comparative anticipatory emotions? We have chosen a straightforward way: by defining their formal counterparts in terms that make them commensurable with

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expected utilities. It is the simplest way to compare experimental choices and the predictions related to our hypothesis that these dual emotions guide our real deliberative choices in uncertainty.

As a benchmark of our tiny formal model, we will take three paradigmatic examples of ‘biases’, first Allais’ paradox, and then two examples of Kahnemann and Tversky’s experiments. In Allais’ paradox, when we consider lottery L1 that offers a better expected utility and L2 giving a certain gain, we prefer the certain gain, but between lotteries L3 and L4 both with moderate probabilities, we prefer that with the higher expected utility.

Although we tend to prefer certainty, this is not always the case: between a lottery with a high stake at a low probability (say a gain of 5000 with probability 0.001) and a surer lottery with a smaller but certain gain (with the same expected utility, say 5 with certainty), we prefer the riskier lottery.

Finally, we will take the example of two presentations, positive and negative, of two therapeutic programmes: one is presented as saving for sure 200 people (among 600) and the other as saving 600 with probability 1/3. Subjects prefer the first programme. But if you tell that it kills 400 people for sure, and that the other kills 600 people with probability 2/3, they prefer the second one.

These biases are inconsistent with the rational choice theory and inconsistent with each other.

In order to compare the branches of two lotteries, we have to consider the states of the world in which we get an outcome, say $G_{\text{max}}$ in lottery L1, and would have got another outcome if we had chosen L2, say $G_{\text{max}}$ in lottery L2. Let us start reasoning on these two independent lotteries, each with only two branches.4 There are four possible states coupling the two outcomes, one from each lottery. Each state is obtained with a probability that is the product of the probability of the two outcomes. In order to simplify the notations, we will write $p_1$ for $p_{\text{max}}^1$ and $p_2$ for $p_{\text{max}}^2$.

$$S_1 = G_{\text{max}}^1 \text{ and } G_{\text{max}}^2 \text{ with probability } p_1 \times p_2.$$ 
$$S_2 = G_{\text{max}}^1 \text{ and } G_{\text{min}}^2 \text{ with probability } p_1 \times (1 - p_2).$$ 
$$S_3 = G_{\text{min}}^1 \text{ and } G_{\text{max}}^2 \text{ with probability } (1 - p_1) \times p_2,$$
$$S_4 = G_{\text{min}}^1 \text{ and } G_{\text{min}}^2 \text{ with probability } (1 - p_1) \times (1 - p_2).$$

In order to explain the formal mechanism, we use as long as possible some relations of symmetry. Elating relief and additional anxiety in gains, additional anxiety and prudential pride in losses respect these relations of symmetry, but prudential pride in gains and relief in losses break them for psychological reasons.

First, we will consider elation. It is raised by the comparison of the unexpected bigger gain $G_{\text{max}}^1$ with the expected smaller one. As expected gain $G_{\text{max}}^1$ has probability $p_1$, we can assign to the unexpected gain $G_{\text{max}}^1$ probability $(1 - p_1)$. The probability of the expected smaller gain is $p_2^1$. The subtraction needed for the comparison cannot be done in the realm of the situations of probability $p_1$, because we focus on the unexpected gain $G_{\text{max}}^1$. It has to be done in the realm of the unexpected gain, in the situations of probability $(1 - p_1)$. We therefore need to multiply the expected smaller gain with probability of the unexpected event $(1 - p_1)^2$. The result of the comparison is given by subtracting (from the unexpected gain $G_{\text{max}}^1$) the smaller gain $G_{\text{max}}^2$ but only in the situation with probability $(1 - p_1) \times p_2^1$.

Second, we will consider anxiety related to the fear that this positive surprise will not occur, the fear of missing the unexpected gain (additional anxiety). We can assign to this missed unexpected gain the probability of the expected gain: $p_2$. In the same way, when evaluating additional anxiety, the comparison will be with the missed expected smaller gain, with probability $(1 - p_2^1)$ instead of $p_2^1$. The subtraction will similarly be done in the realm of $p_1$ situations indicating the missed unexpected event: we subtract $G_{\text{max}}^2 \times p_1^1 (1 - p_2^1)$. Elating relief results from subtracting additional anxiety from elation.

If things were symmetrical between gain and losses, prudential pride would boil down to the comparison of expected utilities. We are proud to be cautious: not to be attracted mainly by higher but riskier gains, but to consider smaller but much more secure gains. For losses, to be cautious consists in not being attracted only by smaller but more probable losses, but also to consider higher but much less probable ones. To be cautious is to try to keep things under control, and in lotteries, probabilities are supposed to be the only sure things in a random world. Expected utility, as a mixture of probability and gain (or loss), may be seen as the ersatz of control that we still have in the realm of randomness. We will see that there is here a bias in gains.

We can summarize the meaning associated with the different probabilities in the following table that refers to bigger gains (losses) only, but it can easily be adapted to smaller gains as well:

<table>
<thead>
<tr>
<th>Gains</th>
<th>Losses</th>
<th>1 - p^1</th>
</tr>
</thead>
<tbody>
<tr>
<td>expected bigger gain</td>
<td>expected avoided bigger loss</td>
<td>p^1</td>
</tr>
<tr>
<td>unexpected bigger gain</td>
<td>unexpected avoided bigger loss</td>
<td>1 - p^1</td>
</tr>
<tr>
<td>missed bigger gain</td>
<td>missed unexpected bigger loss</td>
<td>p^1</td>
</tr>
<tr>
<td>avoided bigger loss</td>
<td>missed anticipated bigger loss</td>
<td>1 - p^1</td>
</tr>
</tbody>
</table>

For losses relief, similar to elation, is related to unexpected avoided bigger loss $L_{\text{max}}^1$ (as we will show, real relief is not the simple counterpart of elation). As with gains $L_{\text{max}}^1 > L_{\text{max}}^2$ and $p^1 < p^2$. By symmetry with elation, we assign to it probability $p^1$. This is because the avoided bigger loss has probability $(1 - p^1)$. Given that this event is unexpected, we associate with the avoided bigger loss probability $p^1$. If things were symmetrical, the comparison would be done with the expected avoided smaller loss $L_{\text{max}}^1$, which implies a probability including a $(1 - p^1)$ factor, in the realm of $p^1$ situations: $p^1 (1 - p^1)$.

Negative additional anxiety, related to missed unexpected avoided loss, will be assigned in the same way as probability $(1 - p^1)$. The comparison will be done with the missed expected avoided smaller loss, with a
probability including a $p^2$ factor, still in the realm of $(1 - p^1)$ situations: $(1 - p^1)p^2$. Finally, prudential pride will be the comparison of expected utilities.

As we have mentioned, there are two biases. First, prudential pride in gains is slightly different. It does not pro-  

Second, relief in losses is not the strict counterpart of elation. Elation is raised by the excitation of risk (and lottery) and prudential pride is biased in gains by this attraction of higher probabilities.

In gains, cautious people are attracted by higher probabilities of gain (even if the gain is smaller than in the other lottery) and prudential pride is biased in gains by this attraction of higher probabilities.

In gains, cautious people are attracted by prudential pride in gains is slightly different. It does not do down to the comparison of expected utilities. In gains, anxious people are attracted by higher probabilities of gain (even if the gain is smaller than in the other lottery) and prudential pride is biased in gains by this attraction of higher probabilities.

In gains, we consider control as less exciting than the possibility that we control in the realm of random. In gains, we may consider control as less exciting than the possibility of a gain. In losses, control is more difficult to disdain.

Nevertheless, we can feel relief in losses! As in contradistinction to elation the contrast between excitement and unexcitement cannot be the basis of relief, the contrast has to be done not between relief and another less anxious state of mind, but between two reliefs involving anxiety, the relief related to the bigger unexpectedly avoided loss and the relief related to the smaller unexpectedly avoided loss.

This requires a comparison with the unexpected avoided smaller loss and not the expected one. We break here the symmetry, and instead of the probability factor $(1 - p^1)$ for expected avoided losses use the factor $(p^1)$ for unexpected avoided losses, always in the realm of $p^1$, that is $(p^1 \times p^2)$. Relief is biased in losses.

As a result of these considerations, we have the following formulae:

- **Prudential pride**
  
  \[
  G^+ = G_{\text{max}}(p^2 - p^1),
  \]
  
  \[
  G^1 = G_{\text{max}}(1 - p^1) - G_{\text{max}}(1 - p^1)p^2,
  \]
  
  \[
  G^2 = (G_{\text{max}} \times p^2 - G_{\text{max}} \times p^2(1 - p^2)),
  \]

- **Additional anxiety**
  
  \[
  L^+ = L_{\text{max}} \times p^1 - L_{\text{max}} \times p^2,
  \]
  
  \[
  L^1 = L_{\text{max}}(p^1) \times L_{\text{max}}(p^2),
  \]
  
  \[
  L^2 = L_{\text{max}}(1 - p^1) - L_{\text{max}}(1 - p^1)^2 \times p^2(1 - p^1).
  \]

In order to check that these definitions are consistent, we make the reasoning visible by representing things on a cube (figure 1). On the $x$-axis, we represent the different gains $G_{\text{max}}$ and $G_{\text{min}}$ of the two lotteries (here, $G_{\text{min}}$ and $G_{\text{min}} = 0$). $G_{\text{max}}$ plays the role of 1 on the $x$-axis. On the $y$-axis, the probabilities of lottery giving $G_{\text{max}}$ are represented and on the $z$-axis, the probabilities of lottery $G_{\text{max}}$. Taking $y$ and not $z$ for the axis of probabilities of $G_{\text{max}}$ would have made the schema more difficult to read. In this way, we can represent the combination of probabilities of the different states (e.g. $S_1 = G_{\text{max}}$ and $G_{\text{max}}$ with probability $p^1 \times p^2 = \text{black, including red}$). Let us take as an example elating relief.

We start considering our representation of getting $G_{\text{max}}$ for sure. This is the whole cube $G_{\text{max}} \times 1 \times 1$ (the $y$-axis, devoted to the probabilities of $L_2$, is taken to be of measure 1; the value ‘1’ will be omitted in the following). As our elation is related to the unexpected gain, we have to subtract the expected gain $G_{\text{max}} \times p^1$: the black and green volumes. We are left with $G_{\text{max}} \times (1 - p^1)$, the situations in which $G_{\text{max}}$ would be unexpected. As it is a comparative elation, we have to take into account the situations in which simultaneously $G_{\text{max}}$ is unexpected and obtained and $G_{\text{max}}$ is expected and obtained. We feel comparative elation only for the unexpected surplus of $G_{\text{max}}$ over the expected other gain. As we have already subtracted the black volume representing $G_{\text{max}} \times p^1$, we have already subtracted the red volume $G_{\text{max}} \times p^2 \times p^2$, which is included in $G_{\text{max}} \times p^2$. We still need to subtract $G_{\text{max}}(1 - p^1)p^2$, the blue volume.

If $p^1_{\text{max}}$ is noted $p^1$ and $p^2_{\text{max}}$ is noted $p^2$ and $G_{\text{min}} = G_{\text{min}} = 0$, then the relevant combinations of gains and probabilities are represented by combinations of coloured volumes as described in figure 1.

Let us apply these formulae to several experimental choices the ‘biases’ of which are each inconsistent with the rational choice theory, as well as being inconsistent with each other.

### 5. EMOTIONAL EXPLANATION OF EXPERIMENTAL ‘BIASES’

1. **Allais’ paradox**

   \[
   L_1 = 2500 \times p^1 = 0.33, \ 2400 \times p^2 = 0.66,
   \]

   \[
   0 \times p^3 = 0.01 \quad \text{and} \quad L_2 = 2400 \times 1.
   \]

   Here three situations have probability 0: those in which we are supposed to get one of the outcomes of $L_1$, but 0 in $L_2$. Three situations are still possible, in

<table>
<thead>
<tr>
<th>comparisons</th>
<th>gains</th>
<th>losses</th>
</tr>
</thead>
<tbody>
<tr>
<td>elation or relief: first expression compared (minus)</td>
<td>unexpected bigger gain: $(1 - p^1)$</td>
<td>unexpected avoided smaller loss: $(1 - p^1)p^2$</td>
</tr>
<tr>
<td>second one</td>
<td>expected smaller gain: $(p^1)$ in the realm of $(1 - p^1)$: $(1 - p^1)p^2$</td>
<td>expected avoided smaller loss: $(1 - p^1)p^2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>symmetry break</td>
</tr>
<tr>
<td></td>
<td></td>
<td>unexpected-asl: $p^1$ instead of $(1 - p^1)p^2$</td>
</tr>
</tbody>
</table>

| additional anxiety: first expression minus | unexpected bigger gain: $(p^1)$ | unexpected avoided smaller loss: $(1 - p^1)$ |
| second one | missed unexpected bigger gain: $(p^1)$ | missed unexpected avoided smaller loss: $(1 - p^1)$ |
| | missed unexpected smaller gain: $(1 - p^1)p^2$ | missed unexpected smaller loss: $(1 - p^1)p^2$ |
| | symmetry break | difference $p^2 - p^1$ |
For the other side of Allais’ paradox, let us take lottery L3: 2500 $\times$ 0.33, 0 otherwise, and lottery L4: 2400 $\times$ 0.34, 0 otherwise.

\[
E = (100(0.34 \times 0.67) = 22.78) \\
+ (2500(0.66 \times 0.67) = 1105.5), \text{ sum } = 1128.33.
\]

\[
AA = -((2500(0.33) = 825) - (2400(0.33 \times 0.66) = 522.72)), \text{ sum } = -302.28.
\]

\[
PP = 2400 \times (0.34 - 0.33) = 24. \text{ This is in favour of L3 and its higher expected utility.}
\]

(ii) The so-called overestimation of small probabilities

L5: 5000 $\times$ 0.001, 0 otherwise and L6: 5 for sure, 

\[
E: 4995 (1 \times 0.999) + 5000 (0.999 \times 0) = 4990, \]

\[
AA: -(5000(0.001) - 5(0.001 \times 0) = -5.
\]

\[
PP = 5 \times 0.999 = 4.995.
\]

As for most of us, the riskier lottery is preferred.

(iii) Let us finally take the example of the two presentations, positive and negative, of two therapeutic programmes, programme A saving for sure 200 persons among 600, programme B saving 1/3 of 600 persons, with their negative side: A kills 400 persons for sure and B has 2/3 chances to kill 600 persons.

Let us first make the evaluation of the positive side

\[
E: 400 \times 1 \times 2/3 + 600 \times (1/3 \times 0) = 400 \times 2/3, \]

\[
AA: 600 \times 1/3 - 200 \times 1/3 \times 0 = 600 \times 1/3 \]

\[
E > AA \text{ by } 200 \times 1/3, \]

PP: 200 $\times$ 2/3.

Programme A wins by 200 $\times$ 1/3.

Now the evaluation of the negative side

\[
R: 600 \times 2/3 - 400 \times 2/3 = 200 \times 2/3 = 400 \times 1/3, \]

\[
AA: 600 \times 1/3 - 400 \times 1/3 = 200 \times 1/3.
\]

No PP, as the expected utilities are the same.

Programme B wins by 200 $\times$ 1/3.

When people consider simultaneously the two presentations, positive and negative, as two faces of the same corner, they take the two programmes as equal. The positive presentation puts the focus on prudential pride and the negative one on relief.5

6. CONCLUSION

If our hypothesis is true, our choices—at least those made in these experimental contexts—stem from the dual emotions of elating relief and prudential pride. Some people, such as gamblers, are more sensitive to elating relief. Some very cautious people, also a minority, are more sensitive to prudential pride, but most of us are guided in our deliberations by a combination of the two.

Might it be possible to design experimental situations in which neuroimagery tests the difference between the two emotions and their activation in deliberations? For example, take the group of people who prefer 5000, 1/1000 over 5 certain. Their neural activations would show more reward activation (striatum) but also more conflict (DLPFC). Take the group of persons who prefer the 5 certain. We would find both less reward activation and less conflict.

We started from diverse experiments, leading in opposite directions. Unfortunately, coherence cannot

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Figure 1. Elation: Expected bigger gain (\(G_{max}^1\)) = \(G_{max}^1 \times p^1\) = Black + Green, Unexpected \(G_{max}^2\) = Violet + Yellow + Blue = \(G_{max}^2 \times (1 - p^1)\), Expected \(G_{max}^2\) = Red + Blue = \(G_{max}^2 \times p^2\), \(G_{max}^2 \times p^2 = \text{Red} = G_{max}^1\) in S1, already subtracted when subtracting expected \(G_{max}^1\) (\(G_{max}^1 \times (1 - p^1)\)\(^2\)) = Yellow: unexpected \(G_{max}^1\) gain in comparison with \(G_{max}^2\) in the realm of \((1 - p^1)\) situations.

\(G_{max}^2 \times p^2(1 - p^1) = \text{Brown} = \text{in S2, expected } G_{max}^2\) in the realm of \((1 - p^1)\), still to be subtracted, Elation = \(G_{max}^1(1 - p^1) - G_{max}^2(1 - p^1)p^2 = \text{Brown} = \text{in S1, already subtracted, Elation = } G_{max}^1(1 - p^1)\)\(^2\)\).

There is no possibility of getting \(G_{sup}^1\) and Gcertain, so that elation is reduced to \((G_{sup}^1 - G_{certain})\) weighted by the probability of the situation in which 0 was expected and not \(G_{sup}^1\): \(p^3\)

\[E = 2500 \times 0.01 = +25.\]

Additional anxiety (AA) focuses on unexpected situations in which we miss \(G_{sup}^1\). The lack of \(G_{sup}^1\) is weighted by the probability of the situation in which we expect to get it (probability \(p^3\)). As Gcertain cannot be missed, we cannot subtract it.

\[AA = -2500 \times 0.33 = -825. \text{ The residual anxiety is } -800. \]

For prudential pride (PP) we have to take into account the cumulated probabilities of 2500 and 2400 in lottery L1. \(PP = 2400 \times (1 - (p^1 + p^2)) = 2400 \times 0.01 = +24 \text{ in favour of L2.}\)

L2 wins by +244. The certain gain of L2 prevails.
be restored by the prospect theory without adding a lot of biases as parameters (e.g. preference for low probabilities). We have tried here to reduce these biases to a minimum of two, while at the same time taking into account testimonies and our personal experience of these mixed emotions. Thus, we have evaluated them in terms of what could be called ‘unexpected utilities’
6, a hypothetical construction designed to take into account all these diverse and apparently incoherent directions. Such a construction cannot be tested in one neuroimaging experiment alone, but most of the behavioural experiments have been already done. Since we have been able to use these different experiments in a converging way to define the constraints and support our hypothesis, we may be able to design various experiments in neuroimaging to test our hypothesis further.

Emotions are much more complex than assumed at the beginning of the current renewal of interest in this field. Neuroimaging has led us to focus on the multidimensionality of affective states in decision-making. Emotional decisions trigger neuronal activity not only in the striatum, insula or amygdala (supposed to be related to pleasure, pain and fear), but also in the prefrontal cortex, in zones related to treatment of conflict and control of execution. Simple decisions raise multidimensional and mixed emotions. These mixed emotions are not beyond the reach of neuroimaging, which will soon be able to give us valuable information on the complex cognitive and affective activities that explain and justify our choices. Here, we have shown how very mixed they are in order to give an explanation of the diverse ‘biases’ of decision and have extracted a rational core (the symmetrical structure) from this multiplicity. The challenge now is to design experiments combining behavioural experimentation and neuroimaging, each devoted to a possible combination of factors, to test the robustness of this symmetrical structure and of its two biases through convergence in the different experiments.

Many thanks to Miriam Teschl for her useful advice and comments, to André Lapied, Alain Leroux, for their comments and critiques, and to one reviewer for suggestions.

ENDNOTES
1 In their seminal paper, Loones & Sugden (1982) define a regret– rejoicing function (regret or rejoicing are represented by the difference between the utilities of two outcomes of different lotteries, when one outcome is supposed to be the case in one lottery and another one could have been got in the other) and make the sum of the products of regret and rejoicing for each comparison between utilities by the probability of each outcome i, for each i. They do not take directly into account the difference between the probabilities of getting outcome i and outcome j. For partial failures of the predictions of the theory, see Loones et al. (1992). They predict violations of monotonicity and equivalence but find only violations of monotonicity.

2 Analysis of regret is still a promising programme of research. Not only may we choose an action because its outcome is neither the worst nor the best one but minimizes regret (Acker 1997), but in a dynamic analysis we may also try to minimize what could be called backward displaced regret when in later periods we focus on the regret remaining from an earlier period in which we have not chosen the best path. We may avoid information about the other possible paths in past period or, on the contrary, later choose paths similar to that which was the best and which we missed during this period in later choices (Krühmer & Stone 2005) (papers pointed out by a reviewer). Nevertheless, a systematic analysis of the effects of regret is still missing. For example, even though we regret our choice for a past period, we may also feel elated if in the following periods we unexpectedly escape the bad consequences of this previous choice and get only their unlikely but outstandingly good outcomes.

3 Striatum is activated not only by rewards, but also by punishments, good outcomes.

Fehr, E. & Gächter, S. 2002 Altruistic punishment in humans. Nature 415, 137–140. (doi:10.1038/415137a)

REFERENCES